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Black holes with scalar hair and asymptotics in $N = 8$ supergravity

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Abstract: We consider $\mathcal{N} = 8$ gauged supergravity in $D = 4$ and $D = 5$. We show one can weaken the boundary conditions on the metric and on all scalars with $m^2 < -\frac{(D-1)^2}{4}+1$ while preserving the asymptotic anti-de Sitter (AdS) symmetries. Each scalar admits a one-parameter family of AdS-invariant boundary conditions for which the metric falls off slower than usual. The generators of the asymptotic symmetries are finite, but generically acquire a contribution from the scalars. For a large class of boundary conditions we numerically find a one-parameter family of black holes with scalar hair. These solutions exist above a certain critical mass and are disconnected from the Schwarzschild-AdS black hole, which is a solution for all boundary conditions. We show the Schwarzschild-AdS black hole has larger entropy than a hairy black hole of the same mass. The hairy black holes lift to inhomogeneous black brane solutions in ten or eleven dimensions. We briefly discuss how generalized AdS-invariant boundary conditions can be incorporated in the AdS/CFT correspondence.

Keywords: AdS-CFT and dS-CFT Correspondence, Black Holes, Supergravity Models.

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1. Introduction

Stationary, asymptotically flat, vacuum black holes in four dimensions are completely characterized by their mass and angular momentum [1] and have horizons of spherical topology [2]. But there appears to be a much richer spectrum of black hole solutions in higher dimensional spacetimes. Recently, a solution of the Einstein vacuum equations in five dimensions was found that describes a rotating black ring with a horizon of topology $S^1 \times S^2$ [3]. Since black rings can carry the same asymptotic charges as the rotating Myers-Perry black holes [4] this means the uniqueness theorem for stationary black holes does not extend to five dimensions.

In higher dimensional spacetimes with compactified extra dimensions, there exist simple vacuum solutions describing black p-branes that are translationally invariant along the compact directions. Gregory and Laflamme (GL) showed these uniform black p-branes are unstable below a critical mass [5]. More recently, a branch of non-uniform black string solutions emerging from the critical uniform string was constructed numerically, first in perturbation theory [6] and then non-perturbatively in [7]. In addition, it has been argued there exists yet another family of static non-uniform black string solutions that are the end state of the decay of unstable uniform strings [8]. These results indicate that the black hole uniqueness theorems do not hold in higher-dimensional, asymptotically flat spacetimes with compactified extra dimensions.

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The GL instability persists with asymptotically anti-de Sitter (AdS) boundary conditions. In [9] the threshold unstable mode was identified for small Schwarzschild-$AdS_5 \times \mathbf{S^5}$ black holes, which are solutions to type-IIB supergravity. This instability was interpreted as signaling a phase transition in the dual gauge theory. Four dimensional AdS-Reissner-Nordstrom black holes, which are solutions of $\mathcal{N} = 8$ gauged supergravity, exhibit a somewhat similar instability precisely when they are locally thermodynamically unstable [10]. This suggests that the black hole uniqueness theorems do not hold in asymptotically anti-de Sitter spacetimes with compactified extra dimensions. One of the objectives of this paper is to demonstrate this explicitely. We will do so by numerically constructing a new class of static black brane solutions in 11-dimensional supergravity compactified on S^7 and in 10-dimensional type-IIB supergravity on S^5.

M-theory on S^7 can be consistently truncated to $\mathcal{N} = 8$ $D = 4$ gauged supergravity in four dimensions [11]. Similarly $\mathcal{N} = 8$ $D = 5$ supergravity is believed to be a consistent truncation of ten dimensional type-IIB supergravity on S^5. Because of the scalar potential introduced by the gauging procedure the maximally supersymmetric vacuum solutions are AdS_4 and AdS_5. Therefore an appealing way to try to find ten or eleven dimensional black brane solutions is to look for asymptotically AdS black holes with scalar hair in the dimensionally reduced supergravity theories.

The original no hair theorem of Bekenstein [12] proves there are no asymptotically flat black hole solutions with scalar hair for minimally coupled scalar fields with convex potentials. This result was extended to the case of minimally coupled scalar fields with arbitrary positive potentials in [13]. Later it was shown [14] there are no hairy, asymptotically AdS black holes where the scalar field asymptotically tends to the true minimum of the potential. In [15], however, an example was given of a hairy black hole where the scalar field asymptotically goes to a negative maximum of the potential. But because the mass of this solution diverges it is not obvious one can regard it as being asymptotically AdS in a meaningful way.

More recently, a one-parameter family of hairy AdS black holes was found in three dimensions [16]. Asymptotically the scalar field again tends to a negative maximum but the potential satisfies the Breitenlohner-Freedman (BF) bound [17]. A careful analysis of the asymptotic solutions revealed they preserve the asymptotic AdS symmetry group [16], despite the fact that the standard gravitational mass diverges. The reason is that the generators of the asymptotic symmetries acquire a contribution from the scalar field, which renders the conserved charges finite. Therefore at least in three dimensional gravity coupled to a single scalar field (with $m^2 = -3/4$), the usual set of AdS-invariant boundary conditions — which corresponds to requiring finite gravitational mass — does not include all asymptotically AdS solutions. The results of [16] indicate there are theories that admit a much larger class of AdS-invariant boundary conditions than those which have been considered so far. This also raises the possibility there is a scalar no hair theorem for some asymptotically AdS boundary conditions and not for others. We investigate these issues in this paper.

Generalized AdS-invariant boundary conditions are studied in section 2. We show that the results of [16] generalize to d-dimensional gravity minimally coupled to a scalar field with arbitrary mass m^2 in the range $-\frac{(d-1)^2}{4} \leq m^2 < -\frac{(d-1)^2}{4} + 1$. In particular we

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show there is a one-parameter family of boundary conditions on the scalar and the metric components that preserve AdS invariance with well defined generators of the asymptotic symmetries. For all AdS-invariant boundary conditions except one or two, the metric as well as the scalar field fall off slower than usual. In section 3 we turn to $\mathcal{N}=8$ $D=5$ gauged supergravity, which contains scalars saturating the BF bound. We first write down the generalized AdS-invariant boundary conditions on the metric components and the scalars. For a large class of AdS boundary conditions we then numerically find a one-parameter family of black hole solutions with scalar hair. When lifted to ten dimensions, these solutions describe electro-vacuum black branes with a perturbed five sphere on the horizon. In section 4 we consider $\mathcal{N}=8$ $D=4$ supergravity and find hairy black holes for generalized AdS boundary conditions on scalars above the BF bound. In section 5, which is reasonably self-contained, we discuss how generalized AdS-invariant boundary conditions can be incorporated in the AdS/CFT correspondence [18]. Finally, in section 6 we summarize our results.

Note: today, the paper [38] appeared on hep-th, which contains results that overlap with some of those presented in section 2 of this paper.

2. Asymptotically AdS spaces with non-localized matter

2.1 Tachyonic scalars in AdS

Recall that if we write AdS_d in global coordinates

$$ds_0^2 = \bar{g}_{\mu\nu}dx^\mu dx^\nu = -\left(1+\frac{r^2}{l^2}\right)dt^2 + \frac{dr^2}{1+r^2/l^2} + r^2 d\Omega_{d-2} \tag{2.1}$$

then for $m^2 < 0$ solutions to $\nabla^2\phi - m^2\phi = 0$ with harmonic time dependence $e^{-i\omega t}$ all fall off asymptotically like

$$\phi = \frac{\alpha}{r^{\lambda_-}} + \frac{\beta}{r^{\lambda_+}} \tag{2.2}$$

where

$$\lambda_\pm = \frac{d-1 \pm \sqrt{(d-1)^2 + 4l^2m^2}}{2} \tag{2.3}$$

The BF bound is

$$m_{BF}^2 = -\frac{(d-1)^2}{4l^2}\,. \tag{2.4}$$

For fields which saturate this bound, $\lambda_+ = \lambda_- \equiv \lambda$ and the second solution asymptotically behaves like $\ln r/r^\lambda$.

To have a definite theory one must impose boundary conditions on the timelike boundary at spacelike infinity. For reflective boundary conditions $\alpha = 0$ it is well known that a scalar field with negative mass squared does not cause an instability in anti de Sitter space, provided that $m^2 \geq m_{BF}^2$ [17, 19]. This is important, since many supergravity theories arising in the low energy limit of string theory contain fields with negative m^2, but

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they all satisfy this bound. For those boundary conditions [20] there is a positive energy theorem [21, 22, 23] which ensures that the total energy cannot be negative whenever this condition is satisfied.

But for $m_{BF}^2 \leq m^2 < m_{BF}^2 + 1$ both solutions (2.3) are normalizable. It has been argued [17, 24, 25] that scalars with masses in this range allow a second AdS-invariant quantization that corresponds to choosing $\beta = 0$ in (2.2). However, it is easy to see this choice cannot define a quantum field theory on the usual AdS background (2.1).

The standard set of boundary conditions on the metric components at spacelike infinity that is left invariant under SO$(d-1,2)$ is given by [26]

$$\begin{aligned} g_{rr} &= \frac{l^2}{r^2} - \frac{l^4}{r^4} + O\left(\frac{1}{r^{d+1}}\right) & g_{tt} &= -\frac{r^2}{l^2} - 1 + O\left(\frac{1}{r^{d-3}}\right) \\ g_{tr} &= O\left(\frac{1}{r^d}\right) & g_{ra} &= O\left(\frac{1}{r^d}\right) \\ g_{at} &= O\left(\frac{1}{r^{d-3}}\right) & g_{ab} &= \bar{g}_{ab} + O\left(\frac{1}{r^{d-3}}\right) \end{aligned} \tag{2.5}$$

where a, b label the angular coordinates on S^{d-2}. A generic asymptotic Killing vector field ξ^μ behaves as

$$\begin{aligned} \xi^r &= O(r) + O(r^{-1})\,, \\ \xi^t &= O(1) + O(r^{-2})\,, \\ \xi^a &= O(1) + O(r^{-2}) \end{aligned} \tag{2.6}$$

and the charges that generate the asymptotic symmetries are given by

$$Q_G[\xi] = \frac{1}{2}\oint dS_i \bar{G}^{ijkl}(\xi^\perp \bar{D}_j h_{kl} - h_{kl}\bar{D}_j \xi^\perp) + 2\oint dS_i \frac{\xi^i \pi^i{}_j}{\sqrt{\bar{g}}} \tag{2.7}$$

where $G^{ijkl} = \frac{1}{2}g^{1/2}(g^{ik}g^{jl} + g^{il}g^{jk} - 2g^{ij}g^{kl})$, $h_{ij} = g_{ij} - \bar{g}_{ij}$ is the deviation from the spatial metric $\bar{g}_{ij}$ of pure AdS and $\bar{D}_i$ denotes covariant differentiation with respect to $\bar{g}_{ij}$.

Consider now a simple time symmetric, spherical test field configuration ϕ on an initial time slice, so that $\phi \ll 1$ everywhere. The coefficient M of the $O(1/r^{d+1})$ correction to the asymptotic behavior of the g_{rr} component is then proportional to the total mass $Q_G[\partial_t]$ of the scalar field configuration. The constraint equation yields

$$M = Q_G[\partial_t] = \frac{(d-2)\pi^{\frac{d-1}{2}}}{2\Gamma\left(\frac{d-1}{2}\right)} \lim_{r\to\infty} \int_0^r \left(m^2\phi^2 + (D\phi)^2\right) \tilde{r}^{d-2} d\tilde{r}\,. \tag{2.8}$$

However one finds this diverges for $\beta = 0$ boundary conditions for all m^2 in the range $m_{BF}^2 \leq m^2 < m_{BF}^2 + 1$, even for arbitrarily small fields. It is, therefore, inconsistent to quantize a test field with these falloff conditions on the standard anti-de Sitter background (2.1). More generally, solutions with $\beta = 0$ boundary conditions on the scalar field cannot be asymptotically anti-de Sitter in the usual sense (2.5). Of course, this does not exclude the possibility that $\beta = 0$ is a valid scalar field boundary condition on a different AdS background, in which the asymptotic behavior of the metric is somehow relaxed whilst preserving the asymptotic AdS symmetry group with well defined generators. This is the subject of the next subsection.

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2.2 Generalized AdS-invariant boundary conditions

We define asymptotically anti-de Sitter spacetimes by a set of boundary conditions at spacelike infinity which satisfy the requirements set out in [26]. We first consider $d \geq 3$ dimensional gravity minimally coupled to a self-interacting massive scalar field with $m^2_{BF} < m^2 < 1 + m^2_{BF}$. We return below to the case in which the BF bound is saturated. The hamiltonian is given by

$$\begin{aligned} H[\xi] &= \int d^{d-1}x \xi^\mu \mathcal{H}_\mu(x) + Q_\phi[\xi] + Q_G[\xi] \\ &= \int d^{d-1}x (\xi^\perp \mathcal{H}_\perp(x) + \xi^i \mathcal{H}_i(x)) + Q_\phi[\xi] + Q_G[\xi] \end{aligned} \tag{2.9}$$

The $\mathcal{H}_\mu$ are the usual hamiltonian and momentum constraints,

$$\begin{aligned} \mathcal{H}_\perp &= \frac{2}{\sqrt{g}}\left(\pi^{ij}\pi_{ij} - \frac{\pi^2}{d-2} + \frac{p^2}{4}\right) + \sqrt{g}\left[-\frac{R}{2} + \frac{1}{2}(D\phi)^2 + V(\phi)\right], \\ \mathcal{H}_i &= -2\sqrt{g} D_j \left(\frac{\pi^j{}_i}{\sqrt{g}}\right) + p D_i \phi\,. \end{aligned} \tag{2.10}$$

where π^{ij} and p are the momenta conjugate to g_{ij} and ϕ. Here we have set $8\pi G = 1$. The requirement that the hamiltonian (2.9) should have well defined functional derivatives determines the variation of the surface integrals [27],

$$\delta Q_\phi[\xi] = -\oint dS_i \delta\phi \left[D^i \phi\, \xi^\perp + \frac{p\xi^i}{\sqrt{g}}\right] \tag{2.11}$$

and

$$\delta Q_G[\xi] = \frac{1}{2}\oint dS_i G^{ijkl}\left(\xi^\perp D_j \delta g_{kl} - \delta g_{kl} D_j \xi^\perp\right) + 2\oint dS_i \frac{\xi^j \delta\pi^i{}_j}{\sqrt{g}} - \oint dS_i \xi^i \frac{\pi^{jk}\delta g_{jk}}{\sqrt{g}}\,. \tag{2.12}$$

To have a definite theory one must impose *some* boundary conditions at spacelike infinity. This means β should generally be some function of α in (2.2). Consider now the class of solutions with the following asymptotic behavior,

$$\phi(r,t,x^a) = \frac{\alpha(t,x^a)}{r^{\lambda_-}} + \frac{f\alpha^{\lambda_+/\lambda_-}(t,x^a)}{r^{\lambda_+}} \tag{2.13}$$

$$\begin{aligned} g_{rr} &= \frac{l^2}{r^2} - \frac{l^4}{r^4} - \frac{\alpha^2 l^2 \lambda_-}{(d-2) r^{2+2\lambda_-}} + O\left(\frac{1}{r^{d+1}}\right) & g_{tt} &= -\frac{r^2}{l^2} - 1 + O\left(\frac{1}{r^{d-3}}\right) \\ g_{tr} &= O\left(\frac{1}{r^{d-2}}\right) & g_{ab} &= \bar{g}_{ab} + O\left(\frac{1}{r^{d-3}}\right) \\ g_{ra} &= O\left(\frac{1}{r^{d-2}}\right) & g_{ta} &= O\left(\frac{1}{r^{d-3}}\right) \end{aligned} \tag{2.14}$$

where f is an arbitrary constant without variation. When $f = 0$ we recover the $\beta = 0$ boundary conditions discussed above. The standard $\alpha = 0$ boundary conditions for localized matter distributions are obtained for $f \to \infty$ (together with $\alpha \to 0$). Remarkably, for

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all values of f this set of boundary conditions preserves the asymptotic anti-de Sitter symmetries. Thus there exists a one-parameter family of AdS-invariant boundary conditions, parameterized by f.

For $f \to \infty$ the asymptotic conditions (2.14) on the metric components reduce to the standard set (2.5). The variation of the gravitational charges $\delta Q_G[\xi]$ is finite in this case, yielding finite conserved charges given by (2.7). The scalar charges are zero, as one expects from localized matter.

On the other hand, for all finite f both $\delta Q_G[\xi]$ and $\delta Q_\phi[\xi]$ diverge like $r^{d-1-2\lambda_-}$. The divergences, however, precisely cancel out. The total charge can therefore be integrated,[1] giving

$$
\begin{aligned}
Q[\xi] &= Q_G[\xi] + \frac{\lambda_-}{2}\oint d\Omega_{d-2}\frac{\xi^\perp}{r} r^{d-1}\left(\phi^2 + \frac{2f(\lambda_+ - \lambda_-)}{d-1}\phi^{\frac{d-1}{\lambda_-}}\right) \\
&= \tilde{Q}_G[\xi] + \frac{2f\lambda_-\lambda_+}{d-1}\oint d\Omega_{d-2}\frac{\xi^\perp}{r} r^{d-1}\phi^{\frac{d-1}{\lambda_-}}
\end{aligned} \tag{2.15}
$$

where $d\Omega_{d-2}$ is the volume element on the unit $d-2$ sphere and $\tilde{Q}_G[\xi]$ is the finite part of the gravitational charge, coming from the standard asymptotic corrections to the AdS metric.

We emphasize again that in the theory defined by $f = 0$ boundary conditions, which is often used in AdS/CFT, one must relax the asymptotic falloff of some metric components to ensure backreaction can be made small and the asymptotic AdS symmetry group is preserved. Although there is no residual finite scalar contribution to the total charges Q in this case, it is only the variation of the sum of both charges that is well defined.

Finally we turn to the case in which the BF bound is saturated. The second independent solution of the linearized scalar field equation now asymptotically behaves like $\ln r/r^\lambda$. The logarithmic component somewhat alters the formulas but there is no essential difference — there is again a one-parameter family of AdS-invariant boundary conditions. Indeed, we find the asymptotic AdS symmetry group is preserved for solutions with the following asymptotic behavior,

$$
\phi = \frac{\alpha}{r^\lambda}\ln r + \frac{\alpha}{r^\lambda}\left(f - \frac{1}{\lambda}\ln\alpha\right) \tag{2.16}
$$

$$
g_{rr} = \frac{l^2}{r^2} - \frac{l^4}{r^4} - \frac{\alpha^2 l^2 \lambda(\ln r)^2}{(d-2)r^{d+1}} - \frac{\alpha^2 l^2(2\lambda f - 1 - 2\ln\alpha)}{(d-2)}\frac{\ln r}{r^{d+1}} + O\left(\frac{1}{r^{d+1}}\right) \tag{2.17}
$$

$$
\begin{aligned}
g_{tr} &= O\left(\frac{1}{r^{d-2}}\right), \qquad g_{ab} = \bar{g}_{ab} + O\left(\frac{1}{r^{d-3}}\right), \qquad g_{ta} = O\left(\frac{1}{r^{d-3}}\right) \\
g_{ar} &= O\left(\frac{1}{r^{d-2}}\right), \qquad g_{tt} = -\frac{r^2}{l^2} - 1 + O\left(\frac{1}{r^{d-3}}\right)
\end{aligned} \tag{2.18}
$$

where $\alpha(t, x^a)$ and f is again an arbitrary constant. For finite f, the variations of the gravitational and scalar charges are logarithmically divergent. But the divergences again

[1]The boundary conditions on π^{ij} are $\pi^{rr} = O(1/r)$, $\pi^{ra} = O(1/r^2)$ and $\pi^{ab} = O(1/r^{d-6-2\lambda_-})$. Hence the third term in eq. (2.12) is zero. The second term in eq. (2.11) also vanishes because $p \sim r^{d-3-2\lambda_-}$.

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cancel out, allowing us to integrate the total charges $Q = Q_\phi + Q_G$. This yields[2]

$$Q[\xi] = \tilde{Q}_G[\xi] + \frac{1}{2}\oint d\Omega_{d-2}\frac{\xi^\perp}{r}\left(\lambda\beta^2 - \alpha\beta + \frac{\alpha^2}{2\lambda}\right) \tag{2.19}$$

where $\beta = \alpha(f - \ln\alpha/\lambda)$.

We will use this expression and (2.15) in the next sections to compute the mass of the hairy black hole solutions we will find.

For $f \to \infty$ we recover the usual falloff conditions on the metric components. Even though the logarithmic mode is switched off in this case, there is still a finite scalar contribution to the conserved charges. This is also evident in the spinorial proof [22] of the positive energy theorem, where the positive Nester mass (which equals $Q[\xi]$) contains an extra scalar contribution [28]. It means the standard gravitational mass that appears in the metric can be negative and need not be conserved during evolution. In other words, for scalar fields saturating the BF bound positivity of the gravitational mass requires boundary conditions that are stronger than those required for finite mass.

3. Hairy black holes in $\mathcal{N} = 8$ $D = 5$ supergravity

3.1 AdS-invariant boundary conditions

$\mathcal{N} = 8$ gauged supergravity in five dimensions [29, 30] is thought to be a consistent truncation of ten dimensional type-IIB supergravity on S^5. The spectrum of this compactification involves 42 scalars parameterizing the coset $E_{6(6)}/\,\mathrm{USp}(8)$. The scalars that are important for our discussion saturate the BF bound and correspond to the subset that parameterizes the coset $\mathrm{SL}(6,\mathbb{R})/\,\mathrm{SO}(6)$. From the higher dimensional viewpoint, these arise from the $\ell = 2$ modes on S^5. The relevant part of the action involves five scalars ϕ_i and takes the form

$$S = \int\sqrt{-g}\left[\frac{1}{2}R - \sum_{i=1}^{5}\frac{1}{2}(\nabla\phi_i)^2 - V(\phi_i)\right] \tag{3.1}$$

where we have set $8\pi G = 1$. The potential for the scalars ϕ_i is given in terms of a superpotential $W(\phi_i)$ via

$$V = \frac{g^2}{4}\sum_{i=1}^{5}\left(\frac{\partial W}{\partial\phi_i}\right)^2 - \frac{g^2}{3}W^2\,, \tag{3.2}$$

W is most simply expressed as

$$W = -\frac{1}{2\sqrt{2}}\sum_{i=1}^{6} e^{2\beta_i} \tag{3.3}$$

[2]The boundary conditions on the momenta are $\pi^{rr} = O(1/r)$, $\pi^{ra} = O(1/r^2)$ and $\pi^{ab} = O(\ln^2 r/r^5)$.

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where the β_i sum to zero, and are related to the five ϕ_i's with standard kinetic terms as follows,

$$\begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \\ \beta_5 \\ \beta_6 \end{pmatrix} = \begin{pmatrix} 1/2 & 1/2 & 1/2 & 0 & 1/2\sqrt{3} \\ 1/2 & -1/2 & -1/2 & 0 & 1/2\sqrt{3} \\ -1/2 & -1/2 & 1/2 & 0 & 1/2\sqrt{3} \\ -1/2 & 1/2 & -1/2 & 0 & 1/2\sqrt{3} \\ 0 & 0 & 0 & 1/\sqrt{2} & -1/\sqrt{3} \\ 0 & 0 & 0 & -1/\sqrt{2} & -1/\sqrt{3} \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \\ \phi_5 \end{pmatrix} . \tag{3.4}$$

The potential reaches a negative local maximum when all the scalar fields ϕ_i vanish. This is the maximally supersymmetric AdS state, corresponding to the unperturbed S^5 in the type-IIB theory. At linear order around the AdS solution, the five scalars each obey the free wave equation with mass

$$m_i^2 = -4 \tag{3.5}$$

which saturates the BF bound (2.4) in five dimensions. Therefore it is trivial to generalize the results of the previous section to include more than one scalar. One finds asymptotic AdS invariance is preserved for solutions with the following asymptotic behavior,

$$\phi_i(r,t,x^a) = \frac{\alpha_i(t,x^a)}{r^2}\ln r + \frac{\alpha_i(t,x^a)}{r^2}\left(f_i - \frac{1}{2}\ln\alpha_i\right) \tag{3.6}$$

$$g_{rr} = \frac{1}{r^2} - \frac{1}{r^4} - \sum_{i=1}^{5}\frac{2\alpha_i^2}{3r^6}(\ln r)^2 - \sum_{i=1}^{5}\frac{2\alpha_i^2}{3r^6}\left(2f_i - \ln\alpha_i - \frac{1}{2}\right)\ln r + O\left(\frac{1}{r^6}\right) \tag{3.7}$$

$$\begin{aligned} g_{tr} &= O\left(\frac{1}{r^3}\right), \qquad g_{ab} = \bar{g}_{ab} + O\left(\frac{1}{r^2}\right), \qquad g_{ta} = O\left(\frac{1}{r^2}\right) \\ g_{ar} &= O\left(\frac{1}{r^3}\right), \qquad g_{tt} = -\frac{r^2}{l^2} - 1 + O\left(\frac{1}{r^2}\right) \end{aligned} \tag{3.8}$$

where $x^a = \chi, \theta, \phi$ and f_i are five constants labelling the different boundary conditions. The charges Q are given by

$$Q[\xi] = Q_G[\xi] + \frac{1}{2}\sum_{i=1}^{5}\oint d\Omega_{d-2}\frac{\xi^\perp}{r}r^{d-1}\left(\lambda\phi_i^2 - \frac{\alpha_i\phi_i}{r^\lambda} + \frac{\alpha_i^2}{2\lambda r^{2\lambda}}\right), \tag{3.9}$$

which is finite for a generic asymptotic Killing vector field.

Nonperturbatively, the five scalars α_i couple to each other and it is generally not consistent to set only some of them to zero. The exception is α_5, which does not act as a source for any of the other fields. In the next section we will consider solutions involving only α_5, so $\alpha_i = 0$, $i = 1, \ldots, 4$. Writing $\alpha_5 = \phi$ and setting $g^2 = 4$ so that the AdS radius is equal to one, the action (3.1) further reduces to

$$S = \int\sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}(\nabla\phi)^2 + \left(2e^{2\phi/\sqrt{3}} + 4e^{-\phi/\sqrt{3}}\right)\right]. \tag{3.10}$$

3.2 Black holes with scalar hair

We now numerically solve the field equations derived from (3.10) to find a class of static, spherically symmetric black hole solutions with scalar hair outside the horizon. Writing the metric as

$$ds_5^2 = -h(r)e^{-2\delta(r)}dt^2 + h^{-1}(r)dr^2 + r^2 d\Omega_3^2 \,. \tag{3.11}$$

The Einstein equations read

$$h\phi_{,rr} + \left(\frac{3h}{r} + \frac{r}{3}\phi_{,r}^2 h + h_{,r}\right)\phi_{,r} = V_{,\phi} \,, \tag{3.12}$$

$$2(1-h) - rh_{,r} - \frac{r^2}{3}\phi_{,r}^2 h = \frac{2}{3}r^2 V(\phi) \,, \tag{3.13}$$

$$\delta_{,r} = -\frac{r}{3}\phi_{,r}^2 \,. \tag{3.14}$$

Regularity at the event horizon R_e imposes the constraint

$$\phi'(R_e) = \frac{R_e V_{,\phi_e}}{2 - 2R_e^2 V(\phi_e)/3} \tag{3.15}$$

Asymptotic AdS invariance requires ϕ asymptotically decays as

$$\phi(r) = \frac{\alpha}{r^2}\left(\ln r - \frac{1}{2}\ln\alpha + f\right) , \tag{3.16}$$

where f is a constant whose value is determined by the boundary conditions. Hence asymptotically

$$h(r) = r^2 + 1 + \frac{2\alpha^2}{3r^2}(\ln r)^2 + \frac{2\alpha^2}{3r^2}\left(2f - \frac{1}{2} - \ln\alpha\right)\ln r - \frac{M_0}{r^2} \,, \tag{3.17}$$

where M_0 is an integration constant.

The Schwarschild-AdS black hole with $\phi = 0$ everywhere is clearly a solution for all boundary conditions. The conserved charge (3.9) reduces to

$$Q[\partial_t] = Q_G[\partial_t] = 3\pi^2 M_0 = 3\pi^2(R_e^4 + R_e^2) \,, \tag{3.18}$$

which is the usual Schwarschild-AdS mass. However, numerical integration of the field equations (3.12)–(3.13) shows that all boundary conditions corresponding to finite f also admit a one-parameter family of static spherically symmetric black hole solutions with scalar hair outside the horizon.

In figure 1 we plot the value ϕ_e of the field at the horizon of the hairy black holes as a function of horizon size R_e. The two curves correspond to solutions with two different AdS-invariant boundary conditions, namely $f = 0$ (bottom) and $f = 1$ (top). Only for $f \to \infty$ we find no regular hairy black hole solutions. For all finite f we find ϕ_e is nonzero for all R_e, even for arbitrarily small black holes. This means the hairy black holes are disconnected from the Schwarschild-AdS solution. In figure 2 we show the hair $\phi(r)$ of a black hole of size $R_e = .2$ that is a solution for boundary conditions corresponding to $f = 1$.

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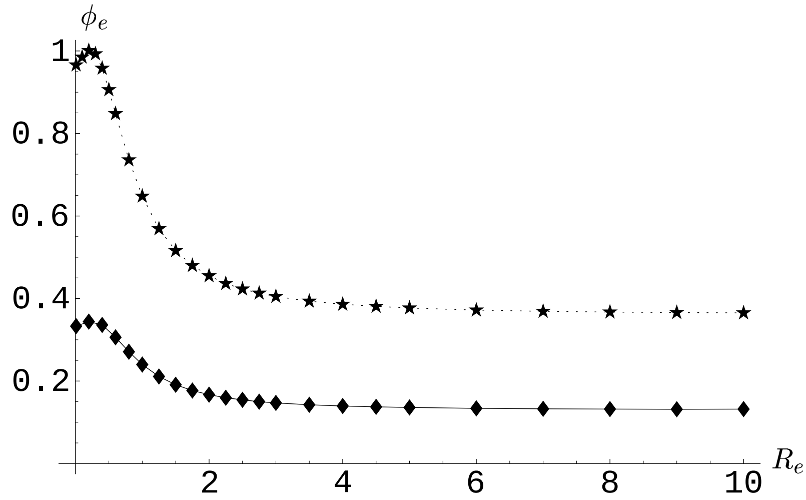

Figure 1: The scalar field ϕ_e at the horizon as a function of horizon size R_e in hairy black hole solutions of $D = 5$ $\mathcal{N} = 8$ supergravity. The two curves correspond to solutions with two different AdS-invariant boundary conditions, namely $f = 0$ (bottom) and $f = 1$ (top).

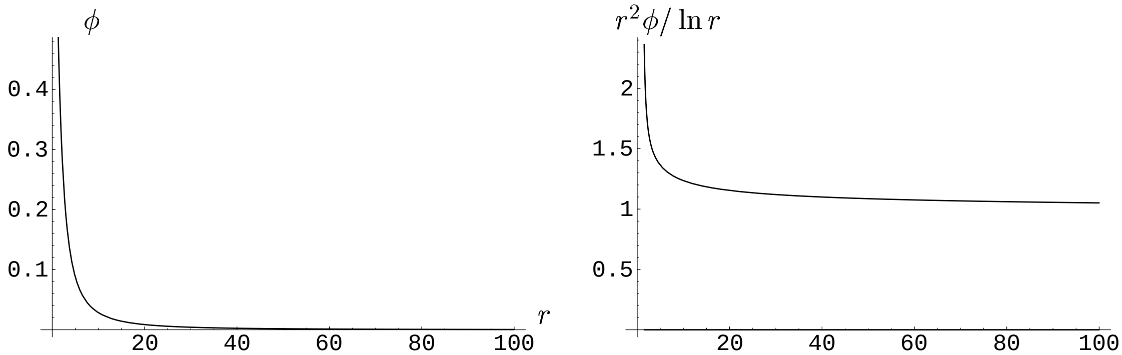

Figure 2: The hair $\phi(r)$ (left) and $r^2\phi/\ln r$ (right) outside a black hole of size $R_e = .2$, with boundary conditions specified by $f = 1$.

The hair $\phi(r)$ decays as $\ln(r)/r^2$ with a $1/r^2$ correction. For given boundary conditions, the coefficient α in (3.16) fully characterizes the asymptotic profile of the hair. Its value is shown in figure 3 for a range of horizon sizes R_e, again for two different boundary conditions $f = 0$ and $f = 1$. One sees that α reaches a minimum at $R_e \approx .2$.

The integration constant M_0 as a function of horizon size R_e is plotted in figure 4. Integrating the constraint equation (3.13) yields the following formal expression for M_0,

$$M_0 = \lim_{r\to\infty}\Bigg[e^{-\frac{1}{3}\int_{R_e}^{r} d\hat{r}\hat{r}(\phi_{,r})^2}(R_e^4 + R_e^2) + \\ + \int_{R_e}^{r} e^{-\frac{1}{3}\int_{\tilde{r}}^{r} d\hat{r}\hat{r}(\phi_{,r})^2}\left[\frac{2}{3}(V(\phi) - \Lambda) + \frac{1}{3}\left(1 + \frac{\tilde{r}^2}{\ell^2}\right)\phi_{,\tilde{r}}^2\right]\tilde{r}^{d-2}d\tilde{r} + \\ + \frac{2\alpha^2}{3}(\ln r)^2 + \frac{2\alpha^2}{3}\left(2f - \ln\alpha - \frac{\alpha}{2}\right)\ln r\Bigg]. \tag{3.19}$$

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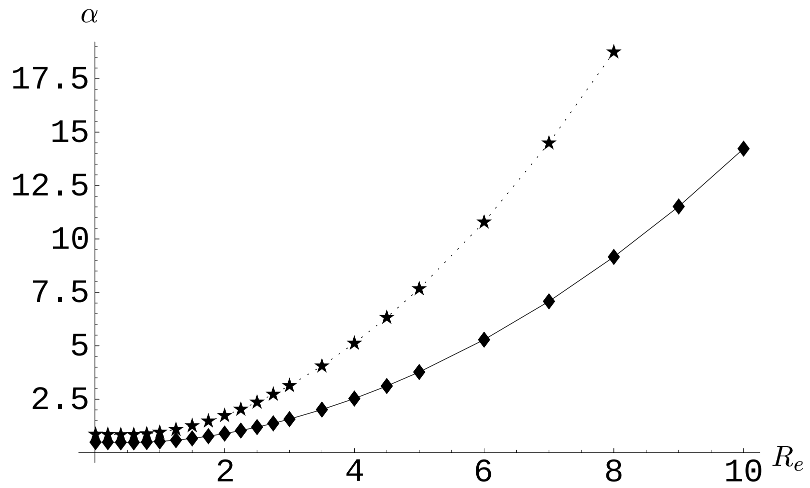

Figure 3: The coefficient α that characterizes the asymptotic profile of the hair $\phi(r)$ as a function of horizon size R_e in hairy black hole solutions of $D = 5$ $\mathcal{N} = 8$ supergravity. The two curves correspond to solutions with two different AdS-invariant boundary conditions, namely $f = 0$ (bottom) and $f = 1$ (top).

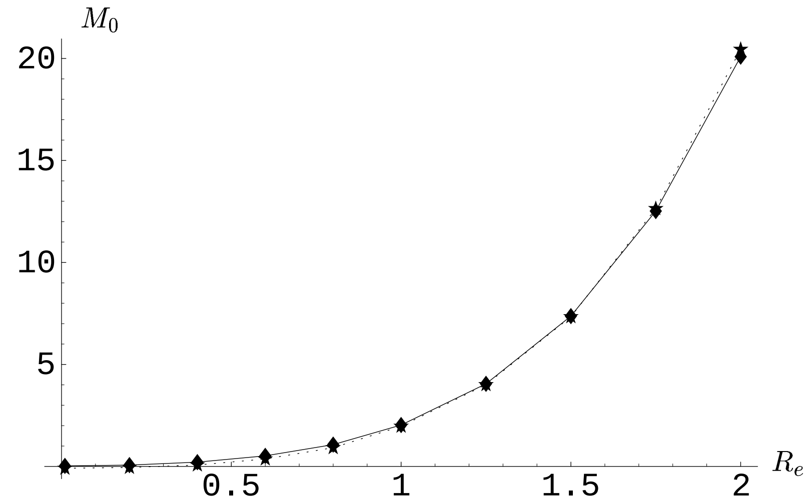

Figure 4: The integration constant M_0 as a function of horizon size R_e in hairy black hole solutions of $D = 5$ $\mathcal{N} = 8$ supergravity. The two curves correspond to solutions with two different AdS-invariant boundary conditions $f = 0$ and $f = 1$ (dotted line).

One sees the hair exponentially 'screens' the Schwarschild-AdS mass. It also introduces new contributions to the gravitational mass which are absent in the Schwarschild-AdS solutions. Figure 4 shows that $M_0 \sim R_e^4$ for large R_e. For small R_e, however, we find $M_0 < 0$, at least with $f = 1$ boundary conditions. As we explained above, this is not in conflict with the positive mass theorem [23] that is believed to hold because this only guarantees the positivity of the conserved charge $Q[\partial_t]$ [28]. The parameter M_0 therefore is of little physical significance. It is proportional to the finite gravitational contribution to $Q[\partial_t]$, but the total gravitational mass diverges. The relevant quantity is the total charge,

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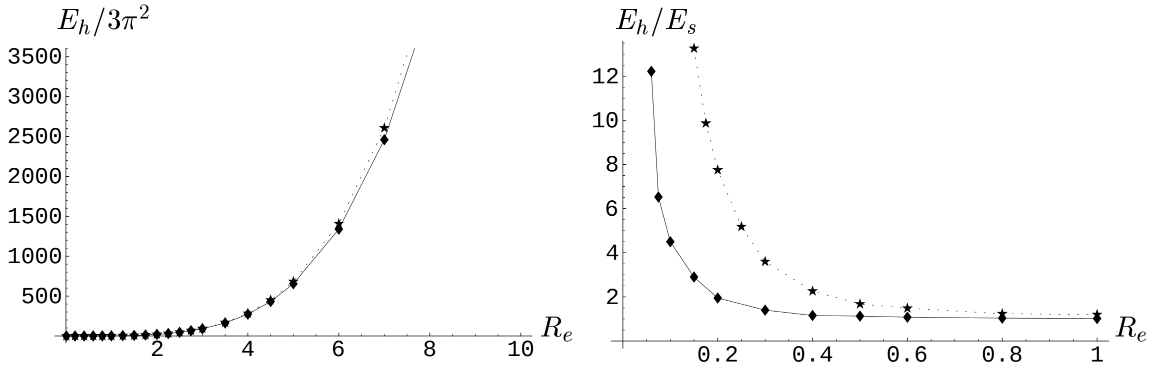

Figure 5: left: The total mass $E_h/3\pi^2$ of hairy black holes as a function of horizon size R_e, in $D = 5$ $\mathcal{N} = 8$ supergravity with two different AdS-invariant boundary conditions $f = 1$ (top) and $f = 0$ (bottom). right: The ratio E_h/E_s as a function of horizon size R_e, where E_s is the mass of a Schwarzschild-AdS black hole of the same size R_e.

which is given by

$$E_h = Q[\partial_t] = 2\pi^2 \left(\frac{3}{2} M_0 + \frac{1}{4}\alpha^2 (\ln \alpha)^2 + \alpha^2 \left(\frac{1}{4} - f \right) \ln \alpha + \alpha^2 \left(f^2 - \frac{1}{2} f + \frac{1}{8} \right) \right). \quad (3.20)$$

The mass E_h is shown in figure 5 as a function of horizon size R_e and for two different boundary conditions $f = 1$ (top) and $f = 0$ (bottom). We find $E_h > 0$ for all R_e and for all boundary conditions we have considered. For large R_e one has $E_h \sim R_e^4$. The mass is also compared with the mass E_s of a Schwarzschild-AdS black hole of the same size R_e. We find $E_h/E_s > 1$ for all R_e and $E_h/E_s \to 1$ for large R_e. Conversely it follows that Schwarzschild-AdS has always larger entropy for a given mass. The ratio E_h/E_s diverges for $R_e \to 0$. We find the hairy black holes can be arbitrarily small, but they only exist above a certain critical mass. The critical mass itself depends on the boundary conditions chosen. At the critical point the solution is nakedly singular.

For fixed AdS-invariant boundary conditions (with finite f), there is precisely one hairy black hole solution for a given total mass $Q[\partial_t]$ (larger than the critical mass). This is because regularity at the horizon (eq. (3.15)) uniquely determines the horizon size and the value of the scalar field at the horizon, for a given mass. This yields a unique combination of α and M_0, which are the two quantities that parameterize the class of static, sperically symmetric asymptotically AdS solutions. Thus we have found a one-parameter family of black holes with scalar hair, in a range of theories parameterized by f. Because Schwarzschild-AdS is a solution too for all boundary conditions we have two very different black hole solutions for a given total mass, one with $\phi = 0$ everywhere and one with nontrivial hair. So the scalar no hair theorem does not hold in supergravity with asymptotically anti-de Sitter boundary conditions. Uniqueness is restored only for $f \to \infty$.

3.3 Black branes in ten dimensions

$D = 5$ $\mathcal{N} = 8$ supergravity is believed to be a consistent truncation of ten dimensional IIB supergravity on S^5. This means that it should be possible to lift our five dimensional

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solutions to ten dimensions. Even though it is not known how to lift a general solution of $D = 5$, $\mathcal{N} = 8$ supergravity to ten dimensions, this is known for solutions that only involve the metric and scalars that saturate the BF bound [31]. So we can immediately write down the ten dimensional analog of the hairy black hole solutions. The ten dimensional solutions involve only the metric and the self dual five form. To describe them, we first introduce coordinates on S^5 so that the metric on the unit sphere takes the form $(0 \leq \xi \leq \pi/2)$

$$d\Omega_5 = d\xi^2 + \sin^2\xi d\varphi^2 + \cos^2\xi d\Omega_3 \,. \tag{3.21}$$

Letting $f = e^{\phi/2\sqrt{3}}$ and $\Delta^2 = f^{-2}\sin^2\xi + f\cos^2\xi$, the full ten dimensional metric is

$$ds_{10}^2 = \Delta ds_5^2 + f\Delta d\xi^2 + f^2\Delta^{-1}\sin^2\xi d\varphi^2 + (f\Delta)^{-1}\cos^2\xi d\Omega_3 \,. \tag{3.22}$$

This metric preserves an $\mathrm{SU}(2) \times \mathrm{U}(1)$ symmetry of the five sphere. The five form is given by

$$G_5 = -U\epsilon_5 - 3\sin\xi\cos\xi f^{-1} * df \wedge d\xi \tag{3.23}$$

where ϵ_5 and $*$ are the volume form and dual in the five dimensional solution and

$$U = -2(f^2\cos^2\xi + f^{-1}\sin^2\xi + f^{-1}) \,. \tag{3.24}$$

One sees that the effect of the hair is to perturb the five sphere on the horizon.

4. Hairy black holes in $\mathcal{N} = 8$ $D = 4$ supergravity

4.1 AdS-invariant boundary conditions

$\mathcal{N} = 8$ $D = 4$ gauged supergravity [32] is the massless sector of the compactification of $D = 11$ supergravity on S^7. We consider the truncation to its abelian $\mathrm{U}(1)^4$ sector. The resulting action is given by

$$S = \int d^4x\sqrt{-g}\left(\frac{R}{2} - \frac{1}{2}\sum_{i=1}^{3}[(\nabla\phi_i)^2 - 2\cosh(\sqrt{2}\phi_i)]\right) + \cdots \tag{4.1}$$

where the dots refer to fields that will be set to zero in our solutions. We have also set $g^2 = 1/4$ so that the curvature scale of anti-de Sitter space is equal to one. The three remaining scalars decouple and they each have mass

$$m^2 = -2 \,, \tag{4.2}$$

which lies in the range $m_{BF}^2 + 1 > m^2 > m_{BF}^2$ in four dimensions.

We will consider solutions whose asymptotic behavior belongs to the following one-parameter class of AdS-invariant boundary conditions,

$$\phi_i(r,t,x^a) = \frac{\alpha_i(t,x^a)}{r} + \frac{f_i\alpha_i^2(t,x^a)}{r^2} \tag{4.3}$$

$$\begin{aligned}
g_{rr} &= \frac{1}{r^2} - \sum_{i=1}^{3}\frac{(1+\alpha_i^2/2)}{r^4} + O\left(\frac{1}{r^5}\right) & g_{tt} &= -r^2 - 1 + O\left(\frac{1}{r}\right)\\
g_{tr} &= O\left(\frac{1}{r^2}\right) & g_{ab} &= \bar{g}_{ab} + O\left(\frac{1}{r}\right)\\
g_{ra} &= O\left(\frac{1}{r^2}\right) & g_{ta} &= O\left(\frac{1}{r}\right)
\end{aligned} \tag{4.4}$$

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where $x^a = \theta, \phi$ and f_i are constants without variation that label the different boundary conditions. The conserved charges $Q = Q_\phi + Q_G$ that generate the asymptotic symmetries are finite and given by

$$Q[\xi] = Q_G[\xi] + \frac{\lambda_-}{2} \sum_{i=1}^{3} \oint d\Omega_{d-2} \frac{\xi^\perp}{r} r^{d-1} \left(\phi_i^2 + \frac{2f(\lambda_+ - \lambda_-)}{d-1} \phi_i^{\frac{d-1}{\lambda_-}} \right) . \tag{4.5}$$

4.2 Black holes with scalar hair

We now look for static, spherically symmetric black hole solutions with scalar hair that are asymptotically AdS. We will concentrate on solutions in which only one scalar $\phi_1 \equiv \phi$ is nonzero. Writing the metric as

$$ds_4^2 = -h(r) e^{-2\delta(r)} dt^2 + h^{-1}(r) dr^2 + r^2 d\Omega_2^2 \tag{4.6}$$

the field equations read

$$h\phi_{,rr} + \left(\frac{2h}{r} + \frac{r}{2} \phi_{,r}^2 h + h_{,r} \right) \phi_{,r} = V_{,\phi} \tag{4.7}$$

$$1 - h - r h_{,r} - \frac{r^2}{2} \phi_{,r}^2 h = r^2 V(\phi) . \tag{4.8}$$

Regularity at the event horizon R_e imposes the constraint

$$\phi'(R_e) = \frac{R_e V_{,\phi_e}}{1 - R_e^2 V(\phi_e)} . \tag{4.9}$$

Asymptotic AdS invariance requires ϕ asymptotically decays as

$$\phi(r) = \frac{\alpha}{r} + \frac{f\alpha^2}{r^2} , \tag{4.10}$$

where f is a given constant that is determined by the choice of boundary conditions. Hence asymptotically

$$h(r) = r^2 + 1 + \frac{\alpha^2}{2} - \frac{M_0}{r} , \tag{4.11}$$

where M_0 is an integration constant.

The Schwarschild-AdS black hole with $\phi = 0$ everywhere outside the horizon is a solution for all AdS-invariant boundary conditions. The mass (4.5) reduces to

$$Q[\partial_t] = 4\pi M_0 = 4\pi (R_e^3 + R_e) , \tag{4.12}$$

which is the standard Schwarschild-AdS mass. However, numerical integration of the field equations (4.7)–(4.8) shows there is a large class of boundary conditions that also admits a one-parameter family of static spherically symmetric black hole solutions with scalar hair outside the horizon.

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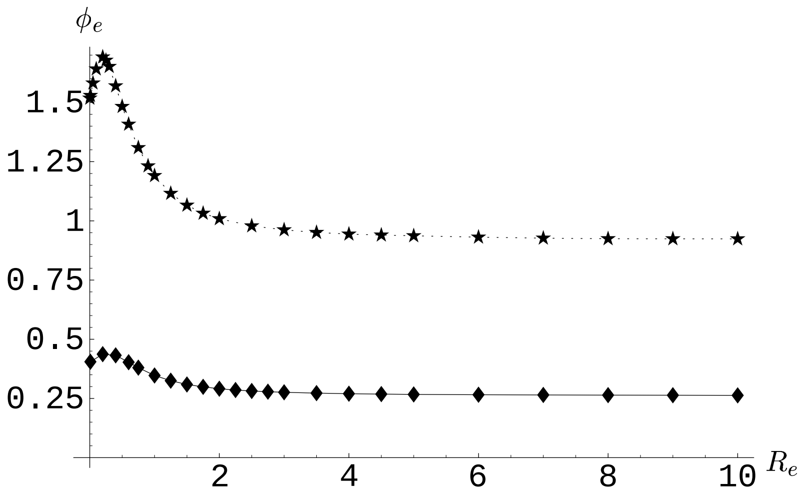

Figure 6: The scalar field ϕ_e at the horizon as a function of horizon size R_e in hairy black hole solutions of $D = 4$ $\mathcal{N} = 8$ supergravity. The two curves correspond to solutions with two different AdS-invariant boundary conditions, namely $f = -1$ (bottom) and $f = -1/4$ (top).

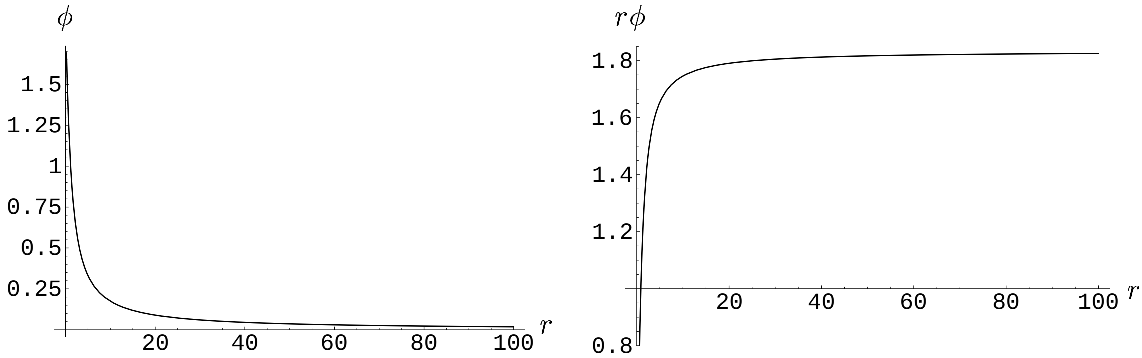

Figure 7: The hair $\phi(r)$ (left) and $r\phi$ (right) outside a black hole of size $R_e = .2$, with boundary conditions specified by $f = -1/4$.

The value ϕ_e of the field at the horizon as a function of horizon size R_e is plotted in figure 6. The two curves correspond to solutions with two different AdS-invariant boundary conditions, namely $f = -1$ (bottom) and $f = -1/4$ (top). Generically, we obtain $\phi_e > 0$ if $f < 0$ and $\phi_e < 0$ for $f > 0$. Only for $f = 0$ and $f \to \infty$ we find no regular hairy black hole solutions. For all finite $f \neq 0$ we find ϕ_e is nonzero for all R_e, even for arbitrarily small black holes. This means the hairy black holes are disconnected from the Schwarschild-AdS solution. In figure 7 we show the hair $\phi(r)$ of a black hole of size $R_e = .2$ that is a solution for boundary conditions corresponding to $f = -1/4$. The hair $\phi(r)$ decays as $1/r$ with a $1/r^2$ correction. For given boundary conditions, the coefficient α in (4.10) fully characterizes the asymptotic profile of the hair. Its value is shown in figure 8 for a range of horizon sizes R_e, again for two different boundary conditions $f = -1$ and $f = -1/4$. One sees that α reaches a (different) positive minimum value at $R_e \approx .2$. For large black holes, we have $\alpha \sim R_e$.

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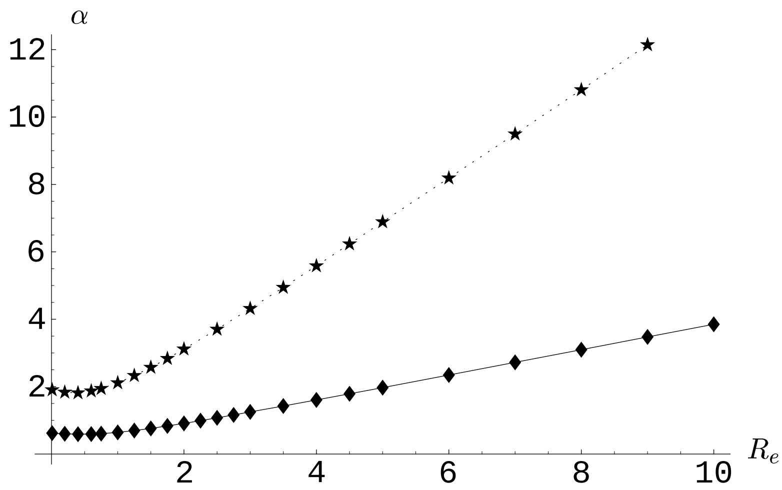

Figure 8: The coefficient α that characterizes the asymptotic profile of the hair $\phi(r)$ as a function of horizon size R_e in hairy black hole solutions of $D = 4$ $\mathcal{N} = 8$ supergravity. The two curves correspond to solutions with two different AdS-invariant boundary conditions, namely $f = -1$ (bottom) and $f = -1/4$ (top).

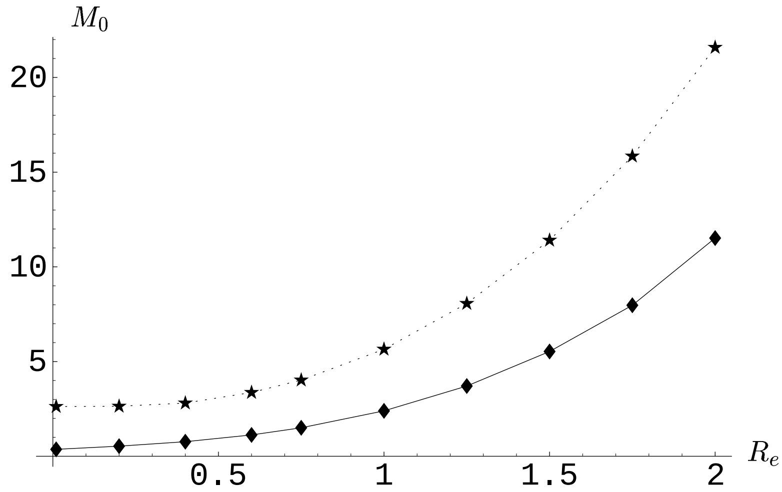

Figure 9: The integration constant M_0 as a function of horizon size R_e in hairy black hole solutions of $D = 4$ $\mathcal{N} = 8$ supergravity. The two curves correspond to solutions with two different AdS-invariant boundary conditions $f = -1$ (bottom) and $f = -1/4$ (top).

The integration constant M_0 as a function of horizon size R_e is plotted in figure 9. We find $M_0 \sim R_e^3$ for large R_e. Integrating the constraint equation (4.8) yields a formal expression for M_0,

$$M_0 = \lim_{r\to\infty}\Bigg[e^{-\frac{1}{2}\int_{R_e}^{r} d\hat{r}\hat{r}(\phi_{,r})^2}(R_e^3 + R_e) + \\ + \int_{R_e}^{r} e^{-\frac{1}{2}\int_{\tilde{r}}^{r} d\hat{r}\hat{r}(\phi_{,r})^2}\left[(V(\phi)+3) + \frac{1}{2}\left(1+\tilde{r}^2\right)\phi_{,\tilde{r}}^2\right]\tilde{r}^2 d\tilde{r} + \frac{\alpha^2}{2}r\Bigg]. \qquad (4.13)$$

One sees the hair exponentially 'screens' the Schwarzschild-AdS mass and introduces new contributions to the gravitational mass which are absent in the Schwarzschild-AdS solutions.

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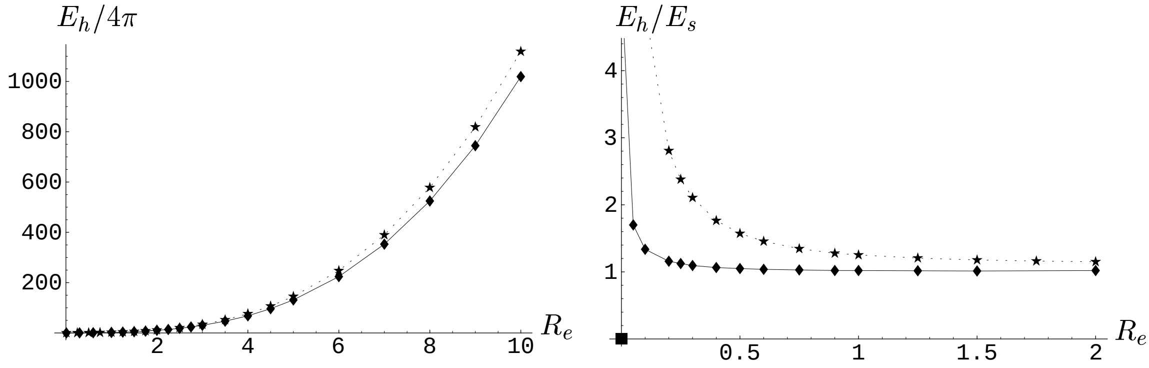

Figure 10: Left: The total mass $E_h/4\pi$ of hairy black holes as a function of horizon size R_e, in $D = 4$ $\mathcal{N} = 8$ supergravity with two different AdS-invariant boundary conditions $f = -1/4$ (top) and $f = -1$ (bottom). Right: The ratio E_h/E_s as a function of horizon size R_e, where E_s is the mass of a Schwarschild-AdS black hole of the same size R_e.

The parameter M_0 is proportional to the finite gravitational contribution to the mass. It is, however, of little physical significance. Indeed the total gravitational mass diverges. The relevant quantity is the total charge $Q[\partial_t]$, which is given by

$$E_h = Q[\partial_t] = 4\pi \left(M_0 + \frac{4}{3} f \alpha^3 \right) . \tag{4.14}$$

The mass E_h is shown in figure 10 as a function of horizon size R_e and for two different boundary conditions $f = -1/4$ (top) and $f = -1$ (bottom). We find $E_h > 0$ for all R_e and for all boundary conditions we have considered. For large R_e one has $E_h \sim R_e^3$. The mass is also compared with the mass E_s of a Schwarschild-AdS black hole of the same size R_e. We find $E_h/E_s > 1$ for all R_e and $E_h/E_s \to 1$ for large R_e. As before, E_h is bounded from below — the hairy black hole solutions exist only above a certain critical mass. The critical mass itself depends on the boundary conditions chosen. At the critical point the solution is nakedly singular.

For fixed AdS-invariant boundary conditions, there is precisely one hairy black hole solution for a given total mass $Q[\partial_t]$. Hence the horizon size as well as the value of the scalar field at the horizon are uniquely determined by $Q[\partial_t]$. Thus we have found a one-parameter family of black holes with scalar hair, in a range of theories parameterized by f. Because Schwarschild-AdS is a solution too for all boundary conditions we have two very different black hole solutions for a given total mass, one with $\phi = 0$ everywhere and one with nontrivial hair. So the scalar no hair theorem does not hold in $D = 4$ $\mathcal{N} = 8$ supergravity with asymptotically anti-de Sitter boundary conditions. Uniqueness is restored only in theories with $f = 0$ or for $f \to \infty$. As before, the hairy black holes can be lifted to black branes in eleven dimensions with a perturbed S^7 on the horizon.

5. AdS/CFT with generalized boundary conditions

We have studied $D = 5$ $\mathcal{N} = 8$ supergravity, which is the low energy limit of string theory with $AdS_5 \times S^5$ boundary conditions, and $D = 4$ $\mathcal{N} = 8$ supergravity, which is

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the low energy limit of string theory with $AdS_4 \times S^7$ boundary conditions. For these boundary conditions, the AdS/CFT correspondence [18] claims string theory is dual to a conformal field theory (CFT). We have shown that the presence of scalars with sufficiently negative m^2 in both supergravity theories allows one to relax the boundary conditions on the metric and on the scalars to include non-localized matter distributions, while preserving the asymptotic AdS symmetry group. According to the general AdS/CFT correspondence, there should be a dual conformal field theory corresponding to each choice of boundary conditions.

Some aspects of AdS/CFT with generalized boundary conditions have already been studied. Let us first consider $D = 4$ $\mathcal{N} = 8$ supergravity, for which the AdS-invariant boundary conditions were given in eqs. (4.3)–(4.4). For simplicity, we consider here generalizing the boundary conditions on a single scalar with $m^2 = -2$. Near the boundary, the field behaves as

$$\phi(t, r, x^a) = \frac{\alpha(t, x^a)}{r} + \frac{\beta(t, x^a)}{r^2} \tag{5.1}$$

where

$$\beta = f\alpha^2 \tag{5.2}$$

together with relaxed falloff conditions (4.4) on the metric.

Because

$$-\frac{(d-1)^2}{4} < m^2 < -\frac{(d-1)^2}{4} + 1 \tag{5.3}$$

it has been argued [24, 25] that ϕ can be associated with CFT operators of two possible dimensions,

$$\Delta_\pm = \frac{(d-1)}{2} \pm \sqrt{\frac{(d-1)^2}{4} + m^2} \tag{5.4}$$

and that supersymmetry constraints are important to decide which assignment is realized in each case. This goes back to the work of [17], where it was argued there are two different AdS-invariant quantizations of a scalar field with m^2 in the range (5.3). For instance for $m^2 = -2$ it is shown [17] one can quantize the scalar field with a boundary condition $\alpha = 0$, in which case it corresponds to an operator $\mathcal{O}'$ of dimension two in the boundary theory, or with a boundary condition $\beta = 0$, in which case it corresponds to an operator $\mathcal{O}$ of dimension one in the boundary theory.

However, this is a rather subtle issue. Indeed we have seen that with $\beta = 0$ boundary conditions (which corresponds to choosing $f = 0$ in (2.13)), one must weaken the falloff of the metric (2.14) in order to preserve asymptotic AdS invariance. This means one does not really have two different quantum field theories on a given AdS background, but two quantum field theories on two different AdS backgrounds. To quantize a test field with $\beta = 0$ on the standard AdS background is inconsistent because the gravitational backreaction would always diverge. By contrast, if one relaxes the asymptotic behavior of the metric, eq. (2.15) automatically yields a finite conserved energy for both choices of boundary conditions. Neglecting backreaction for $f = 0$ boundary conditions then amounts to neglecting the $O(1/r^5)$ correction to the g_{rr}-component in eq. (2.14). Moreover, the two AdS-invariant quantizations we have discussed so far are only two members of a

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one-parameter family of AdS-invariant boundary conditions, parameterized by f. For all finite values of f one must relax the falloff conditions on the metric components to ensure backreaction can be made small and the asymptotic symmetry group is preserved.

This result explains the origin of the extra surface term in [17, 25], which had to be added to the euclidean action to define a finite energy for the 'second' method of quantization. It is concievable that the fact that the metric falls off slower than usual also explains other subtleties encountered in AdS/CFT which have to do with surface terms or normalization factors of correlation functions.

We now make some remarks on the relation between the dual field theories for different f. For the 'standard' boundary condition corresponding to $f \to \infty$ and $\alpha = 0$, AdS/CFT relates ϕ in the boundary theory to a dimension two operator $\mathcal{O}'$ and β is interpreted as its expectation value. Boundary conditions with finite $f = 1/f'^2$ are dual to certain deformations of the original CFT. In particular, Witten has argued [33] they correspond to the addition of a term $W[\mathcal{O}']$, so that after formally replacing $\mathcal{O}'$ by its expectation value β in W one has

$$\alpha = \frac{\delta W}{\delta \beta} . \tag{5.5}$$

For (5.2) this gives

$$W = \frac{2f'}{3} \int d^3x \mathcal{O}'^{3/2} , \tag{5.6}$$

which indeed has the correct dimension to preserve conformal invariance. For $f' \to \infty$, the boundary condition approaches the choice $\beta = 0$ which relates ϕ in the boundary theory to an operator $\mathcal{O}$ of dimension one. Vice versa, the deformation that probes the family of boundary conditions starting with the CFT dual to $\beta = 0$ boundary conditions is

$$W = \frac{f}{3} \int d^3x \mathcal{O}^3 . \tag{5.7}$$

It appears, therefore, that all AdS-invariant boundary conditions given in section 2 can be incorporated in the AdS/CFT correspondence. The dual field theories differ from each other by multi-trace deformations that preserve conformal invariance. Thus we obtain a line of conformal fixed points. In theories with several scalar fields with m^2 in the range (5.3) the different lines of conformal fixed points are parameterized by the dimensionless constants f_i that label the possible bulk boundary conditions.

One expects, however, the change in the asymptotic behavior of the metric for finite f should also deform the dual field theory. So presumably the full deformation should involve the CFT stress tensor as well. This point deserves further study and it may be particularly relevant to shed light on the deformations of $\mathcal{N} = 4$ super Yang-Mills that correspond to generalized boundary conditions in $D = 5$ $\mathcal{N} = 8$ supergravity. In this case, because the scalar fields saturate the BF bound, their asymptotic behavior (3.6) involves a logarithm for finite f. At first sight, it is not clear what could be the dual deformed CFT. From the bulk perspective however, the situation is rather similar to $D = 4$ $\mathcal{N} = 8$ supergravity — each scalar again gives rise to a one-parameter family of AdS-invariant boundary conditions.

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For scalars above the BF bound the total charge (2.15) reduces to the standard gravitational mass for localized matter fields (i.e. $f \to \infty$ boundary conditions). For those boundary conditions it is known a positive mass theorem holds [22]. By contrast, it follows from (3.20) that for scalars saturating the BF bound, there is a (finite) scalar contribution to the total charge (2.19) even if $f \to \infty$. In this case the gravitational mass M_0 can be negative and need not be conserved during evolution [28]. Nevertheless, the positivity of the total charge (2.19) is again ensured by the positive energy theorem [23].

The general proof [22, 23] of the positive energy theorem in asymptotically AdS spaces relies on the existence of asymptotically supercovariant constant spinors. In a supergravity background such spinors — if they exist — will generate asymptotic global supersymmetry transformations. Positivity of the energy is then an immediate consequence of the superalgebra [26]. It is an open question whether our generalized boundary conditions are consistent with asymptotic supersymmetry. From the dual field theory point of view, adding a multitrace interaction like (5.6) or (5.7) breaks supersymmetry. However, because one must also take in account the effect of the weakened metric boundary conditions on the CFT, this issue must be revisited.

6. Discussion

$\mathcal{N} = 8$ supergravity theories in four and five dimensions contain scalar fields with masses in the range $-\frac{(d-1)^2}{4} \leq m^2 < -\frac{(d-1)^2}{4} + 1$. We have shown one can weaken the boundary conditions on the metric and on such scalars to include non-localized matter distributions while preserving the asymptotic AdS symmetry group. The reason is that the divergences of the gravitational charges are cancelled by contributions from the scalars, rendering the total charges finite. We find each scalar with sufficiently negative m^2 gives rise to a one-parameter family of asymptotically AdS boundary conditions in which the metric falls off slower than usual. Generically the generators of the asymptotic symmetries also acquire additional finite contributions from the scalars.

For scalars above the BF bound, the finite scalar contributions vanish for 'localized' matter distributions. Such configurations obey boundary conditions $\phi \sim r^{-\lambda_+}$ (or faster) combined with the standard falloff on all the metric components. In this case, the conserved charge $Q[\partial_t]$ reduces to the standard gravitational mass, which is always finite and positive [22]. For all other AdS-invariant boundary conditions, including $\phi \sim r^{-\lambda_-}$, the scalars do contribute to the conserved charges and the boundary conditions on some metric components must be relaxed in order to preserve the asymptotic AdS symmetry group. The fact that the metric must fall off slower than usual explains for instance the origin of the extra surface terms that are needed in AdS/CFT for these boundary conditions on scalar fields.

Scalars that saturate the BF bound yield finite (positive) contributions to the generators for *all* AdS-invariant boundary conditions. This even includes the usual 'localized' matter distributions where $\phi \sim r^{-(d-1)/2}$ asymptotically, as was pointed out in [28]. The metric has the standard asymptotic behavior for those boundary conditions, but the gravitational mass — which appears in the metric — is generically neither positive nor conserved under evolution.

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For a single scalar, fixed AdS-invariant boundary conditions allow for a two-parameter family of static, sperically symmetric asymptotic solutions. One parameter α characterizes the asymptotic profile of ϕ and a second parameter M_0 (together with α) determines the asymptotic behavior of the metric. Relaxing the falloff on a single scalar in $\mathcal{N} = 8$ supergravity in $D = 4$ and $D = 5$, we found there exists a one-parameter family of AdS black holes with scalar hair for all boundary conditions except a discrete number. The horizon size of the hairy black hole solutions as well as the value of the scalar field at the horizon are uniquely determined by a single charge, namely the total mass $Q[\partial_t]$. The hairy black hole solutions only exist above a critical mass (which depends on the boundary conditions chosen), but their horizon size can be arbitrarily small. Because the Schwarschild-AdS black hole is also a solution for all boundary conditions, one has two very different black hole solutions for a given total mass (above a critical value). Therefore the scalar no hair theorems do not hold in supergravity with asymptotically anti-de Sitter boundary conditions.

For given boundary conditions, we find the scalar field at the horizon of the hairy black holes is always nonzero. Thus the hairy black holes are disconnected from the Schwarschild-AdS solution. We also find the hairy black holes are more massive than Schwarschild-AdS of the same size. The ratio of their masses, however, tends to one for large black holes. Conversely, for a given total mass, the Schwarschild-AdS black hole has always larger entropy.

$\mathcal{N} = 8$ gauged supergravity in four dimensions is a consistent truncation of M-Theory on S^7. Similarly $\mathcal{N} = 8$ gauged supergravity in five dimensions is thought to be a consistent truncation of ten dimensional type-IIB supergravity on S^5. Therefore our hairy black hole solutions can be lifted to new black brane solutions in ten or eleven dimensions. These black branes possess a horizon with a perturbed S^5 or S^7. We should mention, however, that the inhomogeneous black branes cannot be the endstate of the GL instability of Schwarschild-$AdS_5 \times S^5$, because this can only be seen in the dimensionally reduced setup if also the massive spin 2 fields corresponding to higher Kaluza-Klein modes of the metric are included.

The generalized boundary conditions can be incorporated in the AdS/CFT correspondence. Supergravity theories with different asymptotically AdS boundary conditions are dual to different CFT's. The dual field theories differ from each other by certain multi-trace interactions that preserve conformal invariance and by deformations involving the CFT stress tensor. Thus the range of possible boundary conditions on each scalar defines a line of conformal fixed points on the gauge theory side. Whether all AdS-invariant boundary conditions given here are consistent with asymptotic supersymmetry remains an open question.

It would be interesting to study the hairy black holes in the context of the AdS/CFT correspondence. Large Schwarschild-AdS black holes have been conjectured to be described by an approximately thermal state in the gauge theory [34]. The existence of a second black hole solution with the same asymptotic charges poses a puzzle. It suggests there should be some observables in a 'thermal' dual CFT state that are sensitive to the hair.

It would also be interesting to study cosmic censorship [35] in anti-de Sitter space [28, 36, 37] with generalized boundary conditions. In [28] initial data were constructed in $\mathcal{N} = 8$

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$D = 5$ supergravity in which the scalar in (3.10) decays as $\ln r/r^2$ outside a central homogeneous region where $\phi = \phi_0$. A large radius cutoff was imposed to render the gravitational mass finite. The mass of the initial data was then compared with the mass needed to form a black hole large enough to enclose the singularity that develops in the central region. Of course, the cutoff destroys the asymptotic symmetries. However, using our results one can now remove the cutoff and repeat the analysis of [28] while preserving asymptotic AdS-invariance. In addition, one can generalize the analysis to theories and initial data involving scalar fields with different m^2 in the range (5.3). Generically the 'gravitational' mass M_0 will not be conserved during evolution. Instead the relevant quantity to decide whether or not low mass initial data of this type can evolve to black holes is the total conserved charge $Q[\partial_t]$.

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